Chosen-Key Distinguishers on 12-Round Feistel-SP and 11-Round Collision Attacks on Its Hashing Modes

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Abstract. Since Knudsen and Rijmen proposed the *known-key* attacks in ASIA-CRYPT 2007, the *open-key* model becomes more and more popular. As the other component of the *open-key* model, *chosen-key* model was applied to the full attacks on AES-256 by Biryukov *et al.* in CRYPTO 2009. In this paper, we explore how practically the *chosen-key* model affect the real-world cryptography and show that 11-round generic Feistel-SP block cipher is no longer safe in its hashing modes (MMO and MP mode) as there exist collision attacks. This work improves Sasaki and Yasuda's collision attacks by 2 rounds with two interesting techniques. First, we for the first time use the available degrees of freedom in the key to reduce the complexity of the inbound phase, which extends the previous 5-round inbound differential to a 7-round one. This results in a 12-round *chosen-key* distinguisher of Feistel-SP block cipher. Second, inspired by the idea of Wang *et al.*, we construct collisions using two blocks. The *rebound attack* is used in the second compression function. We carefully balance the freedom of the first block and the complexity of the *rebound attack*, and extend the *chosen-key* attack to a 11-round collision attack on its hashing modes (MMO and MP mode).

Keywords: Block Cipher · Feistel-SP · Chosen-Key · Rebound Attack · Hash Mode

Introduction

Nowadays, both block ciphers and hash functions are important primitives in cryptography. In many cases, hash functions are based on block ciphers. For instance, if a block cipher and a hash function are both needed in a resource-restricted environment, such as smart cards, RFID tag, nodes in cars or other machines which work in very tiny embedded systems, many applications utilize a block cipher to construct a hash function in order to minimize the design and implementation cost. There are many popular schemes to construct hash functions based on a block cipher, including the Davies-Meyer(DM), Matyas-Meyer-Oseas(MMO) and Miyaguchi-Preneel(MP) hashing modes, which are all included in the PGV hashing schemes [PGV93]. Thus, the evaluation of the security of block ciphers used in these schemes is very important.

In contrast to the classical block cipher security analysis, which relies on the fact that the key value is kept secret, the key value is known to the attackers in these hashing schemes. Recently, Knudsen and Rijmen [KR07] have proposed to consider the *known-key*

attacks on AES. In their attacks, the key is known and the goal is to find two input messages that satisfy some relations. In a different setting where the key is used as a salt of the hash functions, the key is under the control of the attackers. This attack model is called *chosen-key* model which has been evaluated and popularized by Biryukov *et al.* in [BKN09]. Both models belong to the *open-key* model.

Feistel block cipher adopts an efficient Feistel network design [FNS+75], which is widely trusted and has a long history in cryptography. Historically, many block cipher standards such as DES [Cop94], Triple-DES, MISTY1, Camellia and CAST-128 [Int10] are based on Feistel design. In order to analyze the Feistel primitives comprehensively, Isobe and Shibutani [IS13] classify them into three types, called *Feistel*-1/2/3. In this paper, we focus on the generic *Feistel*-3 type, which has round functions based on substitution-permutation network(SPN), i.e. the round function starts with an XOR of a subkey, followed by a layer of S-Boxes and a linear diffusion layer. We denote *Feistel*-3 as Feistel-SP block cipher in this paper. In ASIACRYPT 2014, Guo *et al.*[GJNS14] gave a 10-round meet-in-the-middle key-recovery attack on generic Feistel-SP block cipher. There are many ciphers adopt Feistel-SP design, such as Camellia [Int10] and Lblock[WZ11], et al. Many key-recovery attacks on these ciphers have been proposed in the last decade, such as [DLJW15, LJWD15], et al. When the Feistel-SP block cipher is used to construct hash function, one needs to analyse its resistance against collision attacks.

Related Work

Knudsen and Rijmen in [KR07] have been the firsts to consider *known-key* distinguishers on AES and Feistel schemes. Besides, they present a half-collision when applying the *known-key* attack to MMO-hashing function with 7-round Feistel block cipher whose round function consists of a round-key XOR followed by an arbitrary key-independent transformation[1]. The main motivations for this model are summarized by paper [DFJ12] as follows:

1. If there is no distinguisher when the key is known, then there will also be no distinguisher when the key is secret;

2. If it is possible to find an efficient distinguisher, finding partial collision on the output of the cipher more efficiently than birthday paradox would predict even though the key is known, then the authors would not recommend the use of such cipher;

3. Finally, such model where the key is known or chosen can be interesting to study the use of cipher in a compression function for a hash function.

In [BKN09], Biryukov *et al.* studied the *chosen-key* distinguisher for the full 256-bit key AES. They showed that in time $q \cdot 2^{67}$, it is possible to construct q-differential multicollisions on Davis-Meyer compression function using AES-256, whereas for an ideal cipher, it would require $q \cdot 2^{\frac{q-1}{q+1}128}$. Then the *chosen-key* distinguisher is translated into a key-recovery attack on the full AES-256 in related-key setting. Lamberger *et al.* [LMR+09] presented a *chosen-key* distinguisher on the full Whirlpool compression function by taking advantage of rebound techniques and the available degrees of freedom of the key. In [NPSS10], Nikolić *et al.* studied the *known-key* and *chosen-key* distinguishers on Feistel and Substitution-Permutation Networks(SPN). So far, many works about *known/chosen-key* models and attacks on ciphers have been proposed, such as [MPP09, KHM+12, ABM13, CS16] *etc.*

At FSE 2011, Sasaki and Yasuda [SY11] obtained an 11-round *known-key* distinguisher by using the rebound attack. Then, they successfully extended the theoretical *known-key* distinguisher to practical collision attacks on hashing mode (MMO and MP mode) with 9-round Feistel-SP block ciphers. In [SEHK12], Sasaki *et al.* studied the *known-key*

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[1]Actually, it is *Feistel*-2 type [IS13], and can be trivially extended to the Feistel-SP block cipher.

scenario for Feistel ciphers like Camellia. Later, Sasaki [Sas12] studied the 4-branch generalised Feistel networks with double SP-functions in *known-key* setting, and he left an open problem in this paper that if *chosen-key* scenario could be applied to the study of the Feistel schemes.

Our Contributions

In this work, we continue to explore how the *open-key* model can impact the real-world cryptography. We give an answer to Sasaki's open problem and show that *chosen-key* scenario works better in the study of the generic Feistel-SP block cipher and its hashing mode. We extend Sasaki and Yasuda's 5-round inbound differential path [SY11] to a 7-round one. Then a 12-round *chosen-key* distinguisher is presented. By exploring the 11-round Feistel-SP block cipher used in MMO/MP-hashing mode, a full-collision attack is constructed which improves Sasaki and Yasuda's collision attacks by 2 rounds. This result shows that the 11-round generic Feistel-SP block cipher is not secure in its hashing mode (MMO and MP mode). It should be noted that our 11-round collision attack is in the same setting as Sasaki and Yasuda's 9-round collision attack, and both of them consider the original hash function's collision(not semi-free-start collision, *etc*). All the results are summarized in Tab. 1, where (N,c)† denotes different cases of Feistel-SP block cipher described in section 1.2 and half-collision‡ means half of bits of difference between two hash values are zero.

Our contributions are three folds:

1. We introduce a new 7-round inbound differential, which extends the Sasaki and Yasuda's inbound path by 2 rounds;

2. We take advantage of the available degrees of freedom in the key to reduce the complexity of the 7-round inbound phase. This is different from the technique used by Lamberger *et al.* [LMR$^+$09]. They also use the available degrees of freedom in the key. However, they use up all the degrees of freedom of the message and the key to make the inbound path hold. In our work, the freedom of the key is used in another way. The 7-round inbound path just consumes the freedom of the message, but the complexity to compute the starting point of the inbound phase is very high. To solve the problem, we use the degrees of freedom of the key and choose some special keys to significantly reduce the time complexity. This results in a 12-round *chosen-key* distinguisher of Feistel-SP block cipher;

3. Inspired by the idea that constructs collision using two blocks [WYY05, WY05], we extend the *chosen-key* distinguishers to collision attacks on MMO and MP hashing modes with 11-round Feistel-SP block ciphers. In our attack, the *rebound attack* is used in the second compression function, where the key is generated by the output of the first compression function, i.e. the chaining value. There is an interesting tradeoff between the degrees of freedom of the chaining value and the time complexity of the *rebound attack*. At last, we connect the two compression functions in the chaining value to produce a 11-round full-collision.

Organization of the Paper

Section 1 gives brief descriptions of Feistel-SP block cipher, some hashing modes and the *rebound attack*. Section 2 presents the related work by Sasaki and Yasuda. Section 3 introduces the new 7-round inbound differential, 12-round *chosen-key* distinguisher on Feistel-SP block cipher and the 11-round full-collision attack on MMO/MP hashing mode. Then other cases of Feistel-SP block ciphers are considered in section 4. In section 5, an experiment is introduced. Finally, we conclude the paper in section 6.

Table 1: Summary of Results for Generic Feistel-SP in Open-Key Mode

Case (N,c)[†]	Rounds	Time	Memory	Power	Source
(128,8)	7	–	–	known-key distinguisher	[BKN09]
	11	2^{19}	2^{19}	known-key distinguisher	[SEHK12]
	12	2^{38}	2^{35}	chosen-key distinguisher	Section 3.2
	7	–	–	half-collision[‡]	[BKN09]
	9	2^{27}	2^{27}	full-collision	[SEHK12]
	11	$2^{48.6}$	2^{27}	full-collision	Section 3.3
(128,4)	7	–	–	known-key distinguisher	[BKN09]
	11	2^{12}	2^{12}	known-key distinguisher	[SY11]
	12	2^{34}	$2^{38.9}$	chosen-key distinguisher	Section 4.1
	7	–	–	half-collision	[BKN09]
	9	2^{24}	2^{24}	full-collision	[SEHK12]
	11	2^{44}	$2^{30.9}$	full-collision	Section 4.1
(64,8)	7	–	–	known-key distinguisher	[BKN09]
	9	2^{19}	2^{19}	known-key distinguisher	[SY11]
	7	–	–	half-collision	[BKN09]
	7	2^{24}	2^{24}	full-collision	[SEHK12]
(64,4)	7	–	–	known-key distinguisher	[BKN09]
	11	2^{11}	2^{11}	known-key distinguisher	[SY11]
	12	2^{18}	2^{19}	chosen-key distinguisher	Section 4.2
	7	–	–	half-collision	[BKN09]
	9	2^{16}	2^{16}	full-collision	[SEHK12]
	11	$2^{24.2}$	2^{15}	full-collision	Section 4.2

1 Preliminaries

In this section, the basic notations used in this paper are introduced. Then we briefly recall the properties of the Feistel block ciphers which are equipped with the SP structures, denoted as Feistel-SP block ciphers. The hashing modes and the *rebound attack* are presented at last.

1.1 Notations

The following notations are used in this paper:

N　　　　The block length of the Feistel-SP cipher (in bits)
n　　　　The size of the input of Feistel-SP cipher's round function, $n = N/2$
c　　　　The size of an S-box in bits
r　　　　The number of S-box sequences, $r = n/c$ in the Feistel-SP cipher
X_i　　　the state after the key addition layer of the ith round
Y_i　　　the state after the substitution transformation layer of the ith round
Z_i　　　the state after the diffusion layer of the ith round
k_i　　　the subkey used in the ith round
$X[i]$　　the ith byte of a bit string X, where the left most byte is $X[1]$
ΔX　　the difference of X and X'
$\oplus$　　　bitwise exclusive OR (XOR)
$|A|$　　　the size of the set A, or the length of a bit string A
$\lll t$　　left circular shift by t bits, e.g. $X \lll t$
$\mathbf{0}$　　　A state where all bytes are non-active

1	A state where only one byte of the prefixed jth position is active
$P(\mathbf{1})$	The output state of the permutation layer, when the input state is **1**
F	A state where all bytes are active

1.2 Feistel-SP Block Ciphers

Isobe and Shibutani [IS13] classify the Feistel block ciphers into three types, called *Feistel*-1/2/3. *Feistel*-3 is also called Feistel-SP block cipher [SY11, SEHK12], which usually adopts 128-bit or 64-bit blocks and use 8-bit or 4-bit S-boxes. As introduced in [SY11, SEHK12], Feistel-SP block ciphers analyzed in this paper are classified as cases (N,c)=(128,8),(128,4),(64,8) and (64,4).
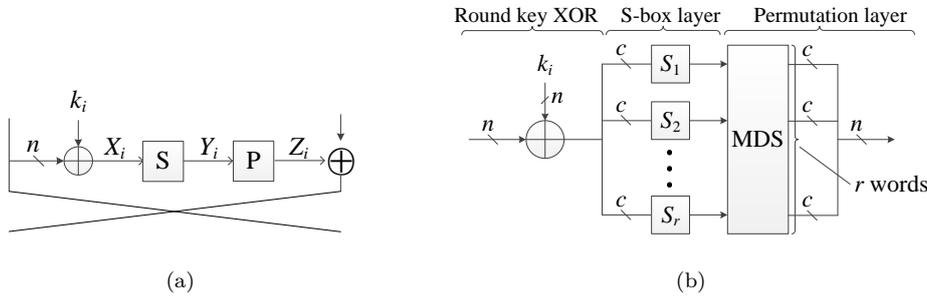

Figure 1: (a) One Round of Feistel-SP block cipher, (b) Detailed Description of the Round Function

As depicted in Fig.1, the round function of Feistel-SP block cipher is composed of the following three operations: .

- **Round key XOR:** The input of the round function is XORed with a round key $k_i \in \mathbb{F}_2^n$.

- **S-box layer(S):** Apply r S-boxes $S_1, S_2, ..., S_r$ in parallel on each c-bit word of the state. All the S-boxes are designed to be resistant to differential and linear cryptanalysis, like the ones used in AES [DR98]. For given two nonzero differences Δ_{in} and Δ_{out} in $\mathbb{F}_2^c$, the equation $S_i(x) \oplus S_i(\Delta_{in} \oplus x) = \Delta_{out}$ has one solution on average.

- **Permutation layer(P):** The linear diffusion, which mixes the values by multiplying an $r \times r$ matrix P, is applied to the output of the S-box layer. Similar to paper [SY11], we also make the assumption that P is a MDS matrix[2], so that the total number of active bytes in the input and output of P is at least $r + 1$, as long as the number of active bytes is not zero.

Key Schedules Assumption[3].

For the four block cipher cases (N,c)=(128,8),(128,4),(64,8) and (64,4), we assume the sizes of their master key K are equal to their state sizes, i.e. $|K| = N$. In this paper, we assume that for a random value $x \in \mathbb{F}_2^n$, there exists a master key that makes $k_5 \oplus k_9 = x$, where k_5, k_9 are round keys of 5th and 9th round generated by the key

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[2]The matching part $\mathbf{1} \to P(\mathbf{1}) \to S \leftarrow P^{-1}(\mathbf{1}) \leftarrow \mathbf{1}$ used in the inbound phase of section 3.1 requires that the active bytes positions in $P(\mathbf{1})$ and $P^{-1}(\mathbf{1})$ are the same, it is not always true if P is not a MDS matrix.

[3]We would like to thank anonymous reviewers of FSE 2016 for reminding us to make this assumption.

schedule[4]. This assumption is weaker than the assumption that the round keys k_5 and k_9 are statistically independent. For example, we denote the master key as $K \in \mathbb{F}_2^{128}$, if $k_5 = (K \lll 1)[1, 2, ..., 8]$ and $k_9 = K[1, 2, ..., 8]$, and obviously k_5 and k_9 are not statistically independent, but for a random given value $x \in \mathbb{F}_2^{64}$, the equation $k_5 \oplus k_9 = x$ always has solutions, which meets our assumption. The key schedules of AES [DR98], Camellia [Int10], CLEFIA [Int11], ARIA [KKP$^+$03], *etc*, all meet our assumption. However, for some lightweight block ciphers, such as LED-128 [GPPR11], Midori64 [BBI$^+$15], where they divide the master key into k_0 and k_1, and use k_0 and k_1 in turn (some round constants will be used to avoid slide attack), our attack does not work.

1.3   Hashing Modes Using Block Ciphers

A hash function is expected to accept almost arbitrary long message inputs. The popular Merkle-Damgård [Dam89, Mer89] domain extension helps us iteratively applying the compression function. Let $f(h_i, m_i)$ denote such a compression function accepting as input a message block m_i and a chaining input h_i, where h_0 is a pre-defined intial value. First, the input message m is padded to be a multiple of the message block length and separated into $m_0 || m_1 || \cdots || m_{L-1}$. Then, all the message blocks are iteratively processed by $h_i = f(h_{i-1}, m_{i-1})$ for $i = 1, 2, \cdots, L$. Finally, h_L is the output as a hash value of m.

In [PGV93], Preneel *et al.* considered a series of compression functions built from a block cipher and proved that 12 modes are secure. Matyas-Meyer-Oseas(MMO) and Miyaguchi-Preneel(MP) modes, which provide efficient ways to construct a compression function from a block cipher, are among the 12 secure schemes. Given a block cipher E and a key K, we denote its encryption algorithm as E_K. The MMO compression function computes h_i by

$$h_i = f(h_{i-1}, (m_{i-1})) = E_{h_{i-1}}(m_{i-1}) \oplus m_{i-1}, \tag{1}$$

where m_{i-1} is a message block and h_{i-1} is the previous chaining value. While the chaining value of the Miyaguchi-Preneel mode is computed by

$$h_i = f(h_{i-1}, (m_{i-1})) = E_{h_{i-1}}(m_{i-1}) \oplus m_{i-1} \oplus h_{i-1}, \tag{2}$$

given m_{i-1} and h_{i-1}.

1.4   The Rebound Attack

The *rebound attack* is a new tool for the cryptanalysis of AES-based hash functions, which was first introduced by Mendel *et al.* in [MRST09]. The main idea is to use the available degrees of freedom in a collision attack to efficiently fulfill the low probability parts in the middle of a truncated differential trail. The rebound attack consists of an inbound phase and a outbound phase depicted in Fig. 2, where W is an internal block cipher or permutation which is split into three subparts, then $W = W_{fw} \circ W_{in} \circ W_{bw}$.

- **Inbound phase:** The inbound phase is a meet-in-the-middle phase in W_{in}. By exploiting the degrees of freedom, the attacker can generate pairs that match the truncated differential path of W_{in} in a low time cost. The matched pairs are denoted as starting points for the outbound phase.

- **Outbound phase:** In this phase, the matched pairs of the inbound phase are computed in both forward and backward direction through W_{fw} and W_{bw} to obtain a pair that satisfy the whole differential path.

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[4]It is because in section 3.1, we need the XOR of some bytes between the subkeys k_5 and k_9 equal some prefixed values.
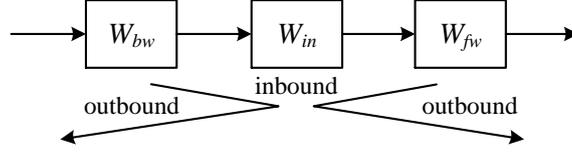
Figure 2: The Rebound-Attack Technique

2 Sasaki and Yasuda's Work

In FSE 2011, by using rebound technique, Sasaki and Yasuda [SY11] introduced 11-round *known-key* distinguisher attacks on different cases of Feistel-SP ciphers and 9-round collision attacks on their hash mode (MMO/MP mode)[5]. All the attacks are based on a 5-round inbound differential path, as follows, depicted in Fig. 3a.

$$(\mathbf{1}, \mathbf{0}) \xrightarrow{4^{th}R} (\mathbf{F}, \mathbf{1}) \xrightarrow{5^{th}R} (\mathbf{0}, \mathbf{F}) \xrightarrow{6^{th}R} (\mathbf{F}, \mathbf{0}) \xrightarrow{7^{th}R} (\mathbf{1}, \mathbf{F}) \xrightarrow{8^{th}R} (\mathbf{0}, \mathbf{1}).$$

Sasaki and Yasuda use the following procedures to find a starting point of the 5-round
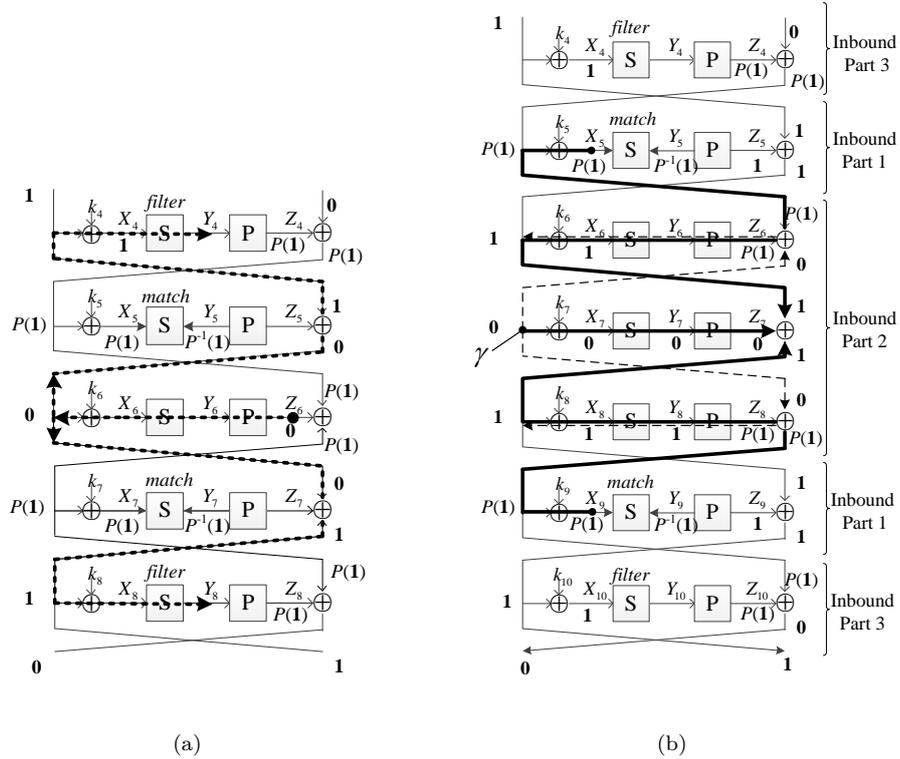

(a)   (b)

Figure 3: (a) Old 5-Round Inbound Phase, (b) New 7-Round Inbound Phase

inbound phase:

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[5]For (N,c)=(64,8), it is a 9-round *known-key* distinguisher attack and 7-round collision attack

1. Prepare the difference distribution tables(DDTs) for all S-boxes. Choose an active-byte position j for differential **1**

2. For all 2^c differences of ΔY_4, compute the corresponding full-byte differences after applying the (forward) permutation layer and store them in a table T. Set the difference in word ΔY_8 to be equal to ΔY_4. This guarantees that the difference in word ΔZ_6 is **0**.

3. For each of the 2^c differences of ΔZ_5, compute the corresponding full-byte difference after applying the inverse permutation. For each difference stored in T, check whether we can match it with the above difference by looking up the DDTs. If a matched set of differences for ΔY_4 and ΔZ_5 is found, we can instantly obtain a matched set for ΔY_8 and ΔZ_7 by setting $\Delta Z_7 = \Delta Z_5$.

4. Now that a matched set of differences is found, we can fix word values and compute the value of word Z_6. Here the values drawn in dashed lines in Fig. 3a are fixed.

   (a) Check whether or not the computed differences in ΔY_4 and ΔY_8 in step 4 and the chosen difference in $\Delta Y_4 = \Delta Y_8$ at step 2 are consistent. Namely, check the following:

   $$\Delta \left[S_j \left(S_j^{-1} \left(\left(P^{-1} \left(Z_6 \right) \right) [j] \right) \oplus k_6 [j] \oplus Z_5 [j] \oplus k_4 [j] \right) \right] \overset{?}{=} \Delta Y_4 \qquad (3)$$

   $$\Delta \left[S_j \left(S_j^{-1} \left(\left(P^{-1} \left(Z_6 \right) \right) [j] \right) \oplus k_6 [j] \oplus Z_7 [j] \oplus k_8 [j] \right) \right] \overset{?}{=} \Delta Y_8 \qquad (4)$$

   (b) If we find a solution for the above two equations, then it means that we have found a starting point of the inbound phase.

Then for cases (N,c)=(128,8),(128,4) and (64,4), the outbound phase for the 11-round *known-key* distinguisher attacks consists of three rounds in backward direction and three rounds in the forward direction. For the 9-round collision attacks, the outbound phase consists of two rounds in backward direction and two rounds in the forward direction. For case (N,c)=(64,8), the attacked rounds are reduced by 2 rounds.

3   New Attacks on Feistel-SP Block Ciphers: Case (N,c)=(128,8)

Similar to the *known-key* setting, the *chosen-key* setting also belongs to the *open-key* model in the literature. In [BKN09], Biryukov *et al.* extended the *chosen-key* distinguisher to a related-key attack on the full AES-256 version, which makes the *chosen-key* model popular. However, these attacks have little effect on the practical use of AES. So one may wonder how practically will these *open-key* models endanger the real-world cryptography. In [SY11], Sasaki and Yasuda introduced *known-key* distinguishers on Feistel-SP block ciphers. Then, they successfully extended the theoretical *known-key* distinguisher to a practical attack by introducing 9-round collisions to its hashing mode (MMO and MP mode). All their attacks are based on a 5-round inbound differential. Later, Sasaki [Sas12] proposed an open problem that if *chosen-key* scenario could be applied to the study of the Feistel schemes.

In this section, we respond to his open problem, and present a *chosen-key* distinguisher on 12-round Feistel-SP block ciphers based on a new 7-round inbound differential, and then we successfully translate the theoretical *chosen-key* distinguisher to a practical attack by giving a 11-round full-collision attack on the MMO and MP hashing modes with these block ciphers, which improves Sasaki and Yasuda's work [SY11] by 2 rounds.

3.1   New 7-Round Inbound Differential

The differential path of the new 7-round inbound phase is

$$(\mathbf{1}, \mathbf{0}) \xrightarrow{4^{th}R} (P(\mathbf{1}), \mathbf{1}) \xrightarrow{5^{th}R} (\mathbf{1}, P(\mathbf{1})) \xrightarrow{6^{th}R} (\mathbf{0}, \mathbf{1}) \xrightarrow{7^{th}R} (\mathbf{1}, \mathbf{0}) \xrightarrow{8^{th}R} (P(\mathbf{1}), \mathbf{1})$$
$$\xrightarrow{9^{th}R} (\mathbf{1}, P(\mathbf{1})) \xrightarrow{10^{th}R} (\mathbf{0}, \mathbf{1}),$$

which is depicted in Fig. 3b.

The 7-round inbound phase is split into 3 parts, i.e. **Inbound Part 1/2/3**. In **Inbound Part 1**, the match-in-the-middle step is applied twice with active bytes $\mathbf{1} \to P(\mathbf{1}) \to S \leftarrow P^{-1}(\mathbf{1}) \leftarrow \mathbf{1}$. In **Inbound Part 2**, we follow the bold lines to formulate Eq. (5) and (6). The matched pairs computed in the match-in-the-middle step will be connected in the middle of this part by calculating γ with Eq. (5) and (6).

$$S^{-1}(\underline{P^{-1}(X_5 \oplus k_5 \oplus \gamma)}) \oplus k_6 \oplus S^{-1}(\underline{P^{-1}(X_9 \oplus k_9 \oplus \gamma)}) \oplus k_8 = P(S(\gamma \oplus k_7)), \quad (5)$$

$$S^{-1}(\underline{P^{-1}(X_5' \oplus k_5 \oplus \gamma)}) \oplus k_6 \oplus S^{-1}(\underline{P^{-1}(X_9' \oplus k_9 \oplus \gamma)}) \oplus k_8 = P(S(\gamma \oplus k_7)). \quad (6)$$

We focus on the ΔX_6 and ΔX_8, where

$$\Delta X_6 = S^{-1}(\underline{P^{-1}(X_5 \oplus k_5 \oplus \gamma)}) \oplus S^{-1}(\underline{P^{-1}(X_5' \oplus k_5 \oplus \gamma)}), \quad (7)$$

$$\Delta X_8 = S^{-1}(\underline{P^{-1}(X_9 \oplus k_9 \oplus \gamma)}) \oplus S^{-1}(\underline{P^{-1}(X_9' \oplus k_9 \oplus \gamma)}). \quad (8)$$

In Eq. (7), if we add(XOR) the underlined formulas, we get $P^{-1}(X_5 \oplus k_5 \oplus \gamma) \oplus P^{-1}(X_5' \oplus k_5 \oplus \gamma) = P^{-1}(X_5 \oplus X_5')$. As shown in **Inbound Part 1** of Fig.3b, $X_5 \oplus X_5'$ is of $P(\mathbf{1})$ differential. Then, $P^{-1}(X_5 \oplus X_5')$ is of $\mathbf{1}$ differential. So, it is easy to know that ΔX_6 is also of $\mathbf{1}$ differential. Similarly, ΔX_8 is of $\mathbf{1}$ differential, too. In our attack, we let all the differentials $\mathbf{1}$ be active in the same byte position. So $\Delta X_6 = \Delta X_8$ happens with probability of 2^{-8}. Once it happens, $S^{-1}(P^{-1}(X_5 \oplus k_5 \oplus \gamma)) \oplus S^{-1}(P^{-1}(X_5' \oplus k_5 \oplus \gamma)) = S^{-1}(P^{-1}(X_9 \oplus k_9 \oplus \gamma)) \oplus S^{-1}(P^{-1}(X_9' \oplus k_9 \oplus \gamma))$ holds. Then, $S^{-1}(P^{-1}(X_5 \oplus k_5 \oplus \gamma)) \oplus k_6 \oplus S^{-1}(P^{-1}(X_9 \oplus k_9 \oplus \gamma)) \oplus k_8 = S^{-1}(P^{-1}(X_5' \oplus k_5 \oplus \gamma)) \oplus k_6 \oplus S^{-1}(P^{-1}(X_9' \oplus k_9 \oplus \gamma)) \oplus k_8$ holds. That means the left two formulas of Eq. (5) and (6) are equal with probability of 2^{-8}.

For a random given master key(round keys k_5, k_6, k_7, k_8, k_9 are computed by the master key) and (X_5, X_5', X_9, X_9'), Eq. (5) is expected to have one solution for γ on average, if we solve γ out, the solution meets Eq. (6) with probability of 2^{-8}(it must meet the condition of $\Delta X_6 = \Delta X_8$). However, Eq. (5) is a nonlinear equation, which is hard to solve, if we calculate γ by exhaustive search, the time complexity is about $2^n = 2^{64}$. Interestingly, we take advantage of the degrees of freedom in the key and elaborately choose some keys to partially linearize Eq. (5) and make it solved much more efficiently .

Collecting keys which help solving Eq. (5) easily.

In Eq. (5), we add(XOR) the underlined two formulas: $P^{-1}(X_5 \oplus k_5 \oplus \gamma) \oplus P^{-1}(X_9 \oplus k_9 \oplus \gamma) = P^{-1}(X_5 \oplus X_9) \oplus P^{-1}(k_5 \oplus k_9)$.

For a given (X_5, X_9), if we find a master key that makes $P^{-1}(k_5 \oplus k_9)[1, 2, 3, 4, 5, 6] = P^{-1}(X_5 \oplus X_9)[1, 2, 3, 4, 5, 6]$, then the underlined two formulas, $P^{-1}(X_5 \oplus k_5 \oplus \gamma)$ and $P^{-1}(X_9 \oplus k_9 \oplus \gamma)$, only differ in bytes 7, 8. Hence, Eq. (5) is simplified as follows,

$$(0, 0, 0, 0, 0, 0, *, *) \oplus k_6 \oplus k_8 = P \circ S(\gamma \oplus k_7), \quad (9)$$

which can be solved by traversing the two unknown bytes $*$, and correspondingly there are 2^{16} possible values of γ. Then we use Eq. (5) to uniquely determine the right connection value γ. The time complexity is 2^{16}. Then the following observation is easy to achieve.

Observation 1. *For a given 64-bit value $P^{-1}(X_5 \oplus X_9)$ and a 6-element-array $1 \leq i_1 \leq i_2 \leq i_3 \leq i_4 \leq i_5 \leq i_6 \leq 8$, there are about $2^{128-48} = 2^{80}$ master keys that form a key set to ensure Eq. (10). There are $\binom{8}{6} = 28$ key sets corresponding to different 6-element-arrays under a given $P^{-1}(X_5 \oplus X_9)$. We denote the union of the 28 key sets as Ukey set labelled by $P^{-1}(X_5 \oplus X_9)$, whose size is about $2^{84.8}$.*

$$P^{-1}(k_5 \oplus k_9)[i_1, i_2, i_3, i_4, i_5, i_6] = P^{-1}(X_5 \oplus X_9)[i_1, i_2, i_3, i_4, i_5, i_6], \qquad (10)$$

where k_5, k_9 are generated by the master key through the key schedule. For a master key in the Ukey set, we can calculate γ with time complexity of 2^{16}.

In observation 1, the *Ukey* set is determined by the 64-bit value $P^{-1}(X_5 \oplus X_9)$. In Alg. 1, in order to check whether a master key belongs to one of such *Ukey* sets, we first compute all the values of X_5, X_9 through all the possible matched difference pairs of $(\Delta X_5, \Delta Y_5)$ and $(\Delta X_9, \Delta Y_9)$, and store all the $P^{-1}(X_5 \oplus X_9)$ values in a table $\mathcal{T}$. For a given master key, we check if there is a 6-element-array $1 \leq i_1 \leq i_2 \leq i_3 \leq i_4 \leq i_5 \leq i_6 \leq 8$ and a value $P^{-1}(X_5 \oplus X_9)$ in $\mathcal{T}$ that ensure Eq. (10). If the we pass the check, the master key belongs to one of such *Ukey* sets, then we can calculate γ efficiently.

After that, we calculate forward and backward to **Inbound Part 3** and the pairs are filtered in this part. At last, the starting points prepared for the outbound phase are generated. The detailed attack procedures are shown in Alg. 1.

Attack Evaluation.

In **Phase A**: it requires $r \cdot 2^{2c}$ computations and $r \cdot 2^{2c}$ memory to prepare r-many DDTs.

- In step (a), there are $r = 8$ possible positions for differential **1**, $j = 1, 2, ...8$.

- In step (b), we are expected to find 2^{2c-r} matches between $\Delta X_5, \Delta Y_5$. Because 2^r solutions of (X_5, X_5') are obtained from a match, we obtain 2^{2c} solutions as long as $2c \geq r$ which is true for case (N,c)=(128,8). It is similar to step (c).

- In step (d), there are about $2^{4c} \times 8 = 2^{35}$ (note there are 8 byte positions for j) values of $(P^{-1}(X_5 \oplus X_9), j, X_5, X_5', X_9')$ needed to be stored in a hash table $\mathcal{T}$.

In **Phase B**:

- In step (ii), every item in table $\mathcal{T}$ corresponds to a different *Ukey* set. If there exists an item in table $\mathcal{T}$ and a 6-element-array $1 \leq i_1 \leq i_2 \leq i_3 \leq i_4 \leq i_5 \leq i_6 \leq 8$, that make Eq. (10) hold, that means the chosen key falls into a *Ukey* set. There are 2^{35} items in $\mathcal{T}$ and each item determines a *Ukey* set. By observation 1, the chosen key falls into one of the 2^{35} *Ukey* sets with probability of $2^{84.8} \times 2^{35} \times 2^{-128} = 2^{-8.2}$.

- In step (iii), the value γ is calculated through Eq. (9) and (5). We traverse the two unknown bytes in Eq. (9) to find a candidate γ and then check it by Eq. (5).

- In step (iv), all the bytes of ΔX_6 and ΔX_8 are zero except one in the same byte position j. This is because, $\Delta X_6 = S^{-1}(P^{-1}(X_5 \oplus k_5 \oplus \gamma)) \oplus S^{-1}(P^{-1}(X_5' \oplus k_5 \oplus \gamma))$, and $P^{-1}(X_5 \oplus k_5 \oplus \gamma) \oplus P^{-1}(X_5' \oplus k_5 \oplus \gamma) = P^{-1}(X_5 \oplus X_5')$ where $\Delta X_5 = X_5 \oplus X_5'$ is of $P(\mathbf{1})$ differential pattern, so $P^{-1}(X_5 \oplus X_5')$ is of $\mathbf{1}$ differential pattern. So ΔX_6 is of $\mathbf{1}$ differential pattern. So as to ΔX_8. Then ΔX_6 equals to ΔX_8 with probability of 2^{-8}.

- In step (v), Eq. (11) and (12) are satisfied with probability of 2^{-16}.

Complexity Evaluation.

In **Phase A**, the time complexity is 2^{35} and memory complexity is 2^{35} 259-bit words to store $\mathcal{T}$.

Algorithm 1 Calculate Starting Point by the 7-round Inbound Phase

Phase A: Prepare DDTs for all S-boxes.

(a) Choose an active-byte position j for differential **1**.

(b) **Inbound Part 1:** For 2^c differences of ΔY_4, compute the corresponding ΔX_5 after applying the (forward) permutation layer. For each of the 2^c differences of ΔZ_5, compute the corresponding full-byte difference ΔY_5 after applying the inverse permutation layer, and check whether ΔX_5 matches ΔY_5 by looking up the DDTs. If we pass the check, go to the following steps.

(c) **Inbound Part 1:** For 2^c differences of ΔY_{10}, compute the corresponding ΔX_9 after applying the (forward) permutation layer. For all 2^c differences of ΔZ_9, compute the corresponding full-byte differences ΔY_9 after applying the inverse permutation layer, and check whether ΔX_9 matches ΔY_9 by looking up the DDTs. If we pass the check, go to the next step.

(d) For the matched pairs $(\Delta X_5, \Delta Y_5)$ and $(\Delta X_9, \Delta Y_9)$, we get values (X_5, X_5'), (X_9, X_9') and store values $(P^{-1}(X_5 \oplus X_9), j, X_5, X_5', X_9')$ in a table $\mathcal{T}$.

Phase B:

(i) Randomly choose a master key, and get all the subkeys by the key schedule.

(ii) Check table $\mathcal{T}$ to determine whether the master key belongs to one of the $Ukey$ sets, if it passes the check, go to the next step; else go to step (i) to choose another master key.

(iii) Calculate γ through Eq. (9) (note that the positions of the two unknown bytes may be changed corresponding to the 6-element-array determined in step (ii).) and Eq. (5).

(iv) Follows the dashed lines, we calculate $\Delta X_6 = S^{-1}(P^{-1}(X_5 \oplus k_5 \oplus \gamma)) \oplus S^{-1}(P^{-1}(X_5' \oplus k_5 \oplus \gamma))$ and $\Delta X_8 = S^{-1}(P^{-1}(X_9 \oplus k_9 \oplus \gamma)) \oplus S^{-1}(P^{-1}(X_9' \oplus k_9 \oplus \gamma))$. If $\Delta X_6 = \Delta X_8$, then go to the next step; else go to step (i) to choose another master key.

(v) Calculate $X_6 = S^{-1}(P^{-1}(X_5 \oplus k_5 \oplus \gamma))$ and X_6', X_8, X_8' similarly. Then calculate $X_4 = k_4 \oplus P(S(X_5)) \oplus X_6 \oplus k_6$ and X_4', X_{10}, X_{10}', similarly. Then check the following two equations. If these two hold, we get a starting point under the chosen key; else go to step (i) to choose another master key.

$$S_j(X_4[j]) \oplus S_j(X_4'[j]) \stackrel{?}{=} \Delta Y_4[j] \tag{11}$$

$$S_j(X_{10}[j]) \oplus S_j(X_{10}'[j]) \stackrel{?}{=} \Delta Y_{10}[j] \tag{12}$$

In **Phase B**, if we choose 2^x different keys, there are $2^{x-8.2}$ left after step (ii), and its time complexity is 2^x table look-ups. The time complexity in step (iii) is equivalent to $2^{x-8.2} \times 2^{16} = 2^{x+7.8}$ 3-round encryptions. The time complexity of step (iv) is $2^{x-8.2}$ and there are $2^{x-8.2-8} = 2^{x-16.2}$ keys left. In step (v), time complexity is $2^{x-16.2}$ and there are $2^{x-16.2-16} = 2^{x-32.2}$ keys left. Finally, we get $2^{x-32.2}$ starting points. For $x = 32.2$, we get one starting point.

Totally, the time complexity is 2^{40} 3-round encryptions which is bounded by step (iii) of the **Phase B**. The memory complexity is 2^{35} 259-bit words to store $\mathcal{T}$. In addition, we need to randomly choose $2^{32.2}$ keys.

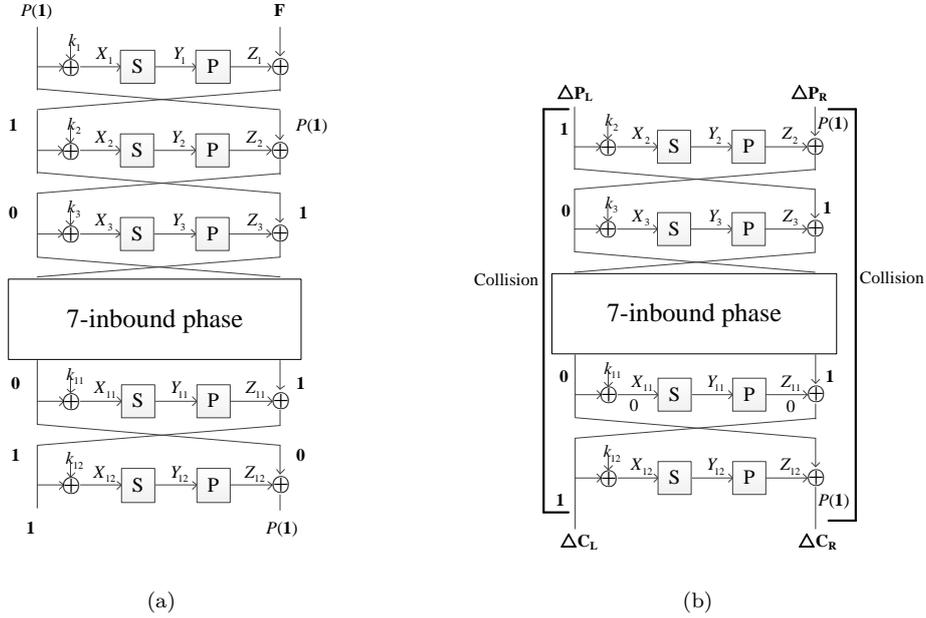## 3.2   12-Round Chosen-Key Distinguisher

Figure 4: (a) 12-round chosen-key distinguisher, (b) 11-round collsion attack

Outbound Phase. As shown in Fig. 4a, we place the 7-round inbound phase between 4 and 10 round. The outbound phase is composed of 3-round backward direction and 2-round forward direction. In the backward, the differential is $(\mathbf{1}, \mathbf{0}) \xrightarrow{3^{th}} (\mathbf{0}, \mathbf{1}) \xrightarrow{2^{th}} (\mathbf{1}, P(\mathbf{1})) \xrightarrow{1^{th}} (P(\mathbf{1}), \mathbf{F})$. In the forward direction, the differential is $(\mathbf{0}, \mathbf{1}) \xrightarrow{11^{th}} (\mathbf{1}, \mathbf{0}) \xrightarrow{12^{th}} (\mathbf{1}, P(\mathbf{1}))$. Hence, when a starting point is given by Alg. 1, the possible output difference of 12th round is limited to $2^{2c} = 2^{16}$ possible values.
Comparison with a Random Permutation.

Given a starting point of the 7-round inbound phase, the outbound phase produces a pair of values that has a differential form of $(P(\mathbf{1}), \mathbf{F})$ for plaintexts and $(\mathbf{1}, P(\mathbf{1}))$ for ciphertexts with probability of 1. So the complexity to obtain such pairs is equal to finding a starting point of the 7-round inbound phases, which is 2^{38} 12-round encryptions.

In the case of random permutation, if we make queries to the encryption oracle, the time complexity is $2^{(N-2c)/2} = 2^{56}$ to find a matching pair. If we make queries to the decryption oracle, according to the limited birthday attack [IPS13], we solve the equation $2^{16 \times 2 - 1 + y} = 2^{56}$, and $y = 25$, then the time complexity is $2^{25+16} = 2^{41}$ 12-round decryptions. Our attack is faster by a factor 2^3.

3.3 11-round Collision Attack

Here we only discuss our collision attacks on the MMO mode, but all the attacks can be trivially extended to the MP mode. This is because the key addition to the hash output state used by the MP mode does not make any impact upon the output value differences.

We add two rounds on the top and two rounds at the bottom of the 7-round inbound path to construct a new 11-round rebound attack, depicted in Fig. 4b. As shown in Fig. 5a, we use two hashing blocks to construct a collision, where the 11-round rebound attack is used in the second compression function. The detail attack procedures are shown in Alg. 2.

Algorithm 2 Generate 11-Round Collision for MMO Hashing Mode with Feistel-SP Block Cipher

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1: Carry out the slightly modified **Phase A** of Alg. 1, where the step (d) of **Phase A** is modified as follows: We pick out the matched pairs $(\Delta X_5, \Delta Y_5)$ and $(\Delta X_9, \Delta Y_9)$ under the condition of $\Delta Y_5 = \Delta Y_9$, and get values (X_5, X_5'), (X_9, X_9') and store values $(P^{-1}(X_5 \oplus X_9), j, X_5, X_5', X_9')$ in a table $\mathcal{T}_1$.
2: As shown in Fig. 5b, randomly choose 2^x values M_0, compute H_1. Carry out the **Phase B** of Alg. 1, where 2^x values of H_1 work as the chosen keys, and calculate the starting points of the 7-round inbound phase.
3: Then we turn to the outbound phase and calculate two rounds forward and two rounds backward. If we get a collision in H_2, the whole attack stops.
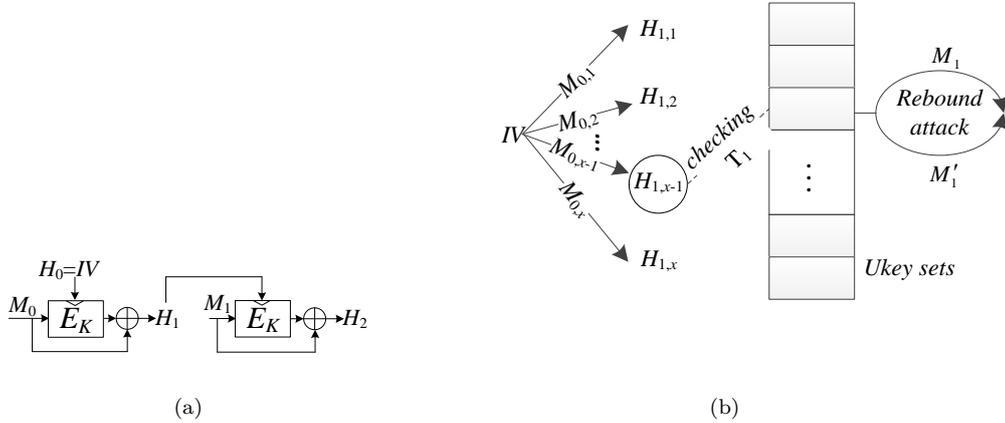

(a)   (b)

Figure 5: (a) MMO Hashing Mode with Two Blocks, (b) The Whole Collision Attack Structure

Attack Evaluation. The whole collision attack is shown in Fig. 5b, we give a brief evaluation as follows.

In the second compression function, as shown in Fig. 4b, we denote the difference of the left and right branch of plaintext as $(\Delta P_L, \Delta P_R)$, and denote the difference of the left and right branch of ciphertext as $(\Delta C_L, \Delta C_R)$. We get a collision in H_2 if and only if we get a pair that $\Delta P_L = \Delta C_L$ and $\Delta P_R = \Delta C_R$.

Step 1 is slightly different from **Phase A** of Alg. 1. For each item in table $\mathcal{T}_1$, $\Delta Y_5 = \Delta Y_9$, and then $\Delta Z_5 = \Delta Z_9$. The size of table $\mathcal{T}_1$ is reduced to $2^{35-8} = 2^{27}$.

In step 2, we carry out **Phase B** of Alg. 1 and generate pairs satisfying $\Delta X_6 = \Delta X_8$. From the inbound differential, it has that $\Delta Z_5 \oplus \Delta Z_9 \oplus \Delta X_6 \oplus \Delta X_8 = \Delta Z_5 \oplus \Delta Z_9 = \Delta P_L \oplus \Delta C_L$. Since all pairs are picked out in table $\mathcal{T}_1$ and satisfy $\Delta Z_5 = \Delta Z_9$. So $\Delta P_L \oplus \Delta C_L = 0$. In other words, the starting points computed in step 2 of Alg. 2 satisfy $\Delta P_L \oplus \Delta C_L = 0$.

In step 3, we only need to check whether ΔP_R equals to ΔC_R or not. If $\Delta P_R = \Delta C_R$, then we get a collision at H_2. So 2^8 starting points are needed from step 2.

Complexity Evaluation.

In step 1, the memory cost is 2^{27} 259-bit words to store $\mathcal{T}_1$.

In step 2, we choose $x = 48.2$. Note that, in the complexity evaluation of the **Phase B** of Alg. 1, we can get one starting point using $2^{32.2}$ chosen keys when the size of $\mathcal{T}$ is 2^{35}. And when the size is reduced to 2^{27}, that means we will filter out more keys. We must use more chosen keys to find a starting point, i.e. we need $2^{32.2+8} = 2^{40.2}$ chosen keys. Finally, we get $2^{48.2-(32.2+8)} = 2^8$ starting points.

In section 3.1, the complexity evaluation of Alg. 1 shows that it costs 2^{40} 3-round encryptions to find a starting point, which is bounded by step (iii) of **Phase B**, and $2^{32.2}$ chosen keys are needed. In step 2 of Alg. 2, we need $2^{40.2}$ chosen keys to find a starting point which increases by a factor of 2^8 compared with $2^{32.2}$ chosen keys. However, it filter out a factor of $2^{8.2+8} = 2^{16.2}$ keys after step (ii) of **Phase B** by using $\mathcal{T}_1$, while $\mathcal{T}$ only filter out a factor of $2^{8.2}$. Then the time complexity of step 2 to find one starting point bounded by step (iii) of **Phase B** is $2^{40.2-16.2+16} = 2^{40}$ 3-round encryptions or $2^{38.13}$ 11-round encryptions. Totally, the time complexity of step 2 is $2^{48.2} + 2^{38.13+8} = 2^{48.61}$ 11-round encryptions. The time complexity of step 3 is 2^8.

Totally, the time complexity to find a collision in H_2 is $2^{48.61}$ 11-round encryptions. The memory complexity dominated by step 1 is 2^{27} 259-bit words. In addition, $2^{48.2}$ values of M_0 are needed. This is faster than the generic birthday bound 2^{64} for collision attack.

4   Attacks on Other Cases (N,c)=(128,4),(64,8) and (64,4)

4.1   Attacks: Case (N,c)=(128,4)

For case (N,c)=(128,4), we modify the inbound phase as

$$(\mathbf{2},\mathbf{0}) \xrightarrow{4^{th}} (P(\mathbf{2}),\mathbf{2}) \xrightarrow{5^{th}} (\mathbf{2},P(\mathbf{2})) \xrightarrow{6^{th}} (\mathbf{0},\mathbf{2}) \xrightarrow{7^{th}} (\mathbf{2},\mathbf{0}) \xrightarrow{8^{th}} (P(\mathbf{2}),\mathbf{2})$$
$$\xrightarrow{9^{th}} (\mathbf{2},P(\mathbf{2})) \xrightarrow{10^{th}} (\mathbf{0},\mathbf{2}).$$

Modify the observation 1: choose an 13-element-array $1 \le i_1 \le i_2 \le i_3 \cdots \le i_{11} \le i_{12} \le i_{13} \le 16$ to construct a master key set that make Eq. (13) hold for a given $P^{-1}(X_5 \oplus X_9)$. There are $\binom{16}{13} = 2^{9.1}$ different key sets, whose union is also denoted as $Ukey$ set. Hence, the size of $Ukey$ is $2^{128-52+9.1} = 2^{85.1}$ for a given $P^{-1}(X_5 \oplus X_9)$.

$$P^{-1}(k_5 \oplus k_9)[i_1, i_2, \cdots, i_{12}, i_{13}] = P^{-1}(X_5 \oplus X_9)[i_1, i_2, \cdots, i_{12}, i_{13}] \tag{13}$$

Similar to Alg. 1, the match-in-the-middle step is applied twice, then we get $2^{8 \times 4} \times C_{16}^2 = 2^{38.9}$ 274-bit values $(P^{-1}(X_5 \oplus X_9), i, j, X_5, X_5', X_9')$ stored in a table $\mathcal{T}'$, where i, j are active nibble positions of differential $(\mathbf{2},\mathbf{0})$ and $1 \le i < j \le 16$. Then by applying **Phase B** of Alg. 1, we find a starting point by using 2^{28} random chosen keys, the time complexity is 2^{36} 3-round encryptions(or 2^{34} 12-round encryptions) and the total memory cost is $2^{38.9}$ 274-bit words.

The differential of the 12-round *chosen-key* distinguisher is

$$(P(\mathbf{2}),\mathbf{F}) \xrightarrow{1^{th}} (\mathbf{2},P(\mathbf{2})) \xrightarrow{2^{th}} (\mathbf{0},\mathbf{2}) \xrightarrow{3^{th}} (\mathbf{2},\mathbf{0}) \xrightarrow{Inbound} (\mathbf{0},\mathbf{2}) \xrightarrow{11^{th}} (\mathbf{2},\mathbf{0}) \xrightarrow{12^{th}} (\mathbf{2},P(\mathbf{2})).$$

The complexity to find a pair that matches the output differential $(\mathbf{2}, P(\mathbf{2}))$ is equal to finding a starting point of the 7-inbound phase, obviously it is faster than the generic limited-birthday bound 2^{41} 12-round encryptions.

The 11-round collision attack is similar to Alg. 2, the differential of the rebound attack starts from the second round of the 12-round differential. The time complexity is about 2^{44} 11-round encryptions, the memory cost is $2^{30.9}$ 274-bit words, and 2^{44} messages are needed. While, the generic birthday bound is 2^{64}.

4.2   Attacks: Cases (N,c)=(64,4) and (N,c)=(64,8)

Case (N,c)=(64,4) is similar to (128,8), the time complexity of 12-round *chosen-key* distinguisher is 2^{18} 12-round encryptions, the memory cost is 2^{19} 131-bit words, $2^{12.2}$ random chosen keys are needed. While the generic limited-birthday bound is 2^{21} 12-round encryptions.

For 11-round collision attack, the time complexity is $2^{24.2}$ 11-round encryptions, the memory cost is 2^{15} 131-bit words, $2^{20.2}$ messages are needed. While the generic birthday bound is $2^{64/2} = 2^{32}$.

In the case (N,c)=(64,8), the time to find a starting point of the 7-round inbound phase is about 2^{32}, which is not faster than the birthday complexity. So the 7-round inbound phase can not be used in this case.

5   Experiment For Case (N,c)=(128,8)

Due to the lack of Feistel-SP block cipher for case (N,c)=(128,8), we modify Camellia [Int10] block cipher for experiment. As we know, the linear permutation of Camellia is not a MDS matrix, so we replace it by the MDS matrix P showed in Eq. (14) borrowed from block cipher Khazad [BR00], then we remove the FL/FL^{-1} layer and make the other modules of Camellia unchanged. Note that the linear permutation in the key schedule is also replaced by the MDS matrix. We call the new block cipher as Camellia-MDS, whose C++ code is listed in the full version of this paper [DW16]. Our experiment works on 12-round reduced Camellia-MDS with 128-bit key.

$$
P = \begin{pmatrix}
0\text{x}01 & 0\text{x}03 & 0\text{x}04 & 0\text{x}05 & 0\text{x}06 & 0\text{x}08 & 0\text{x}0B & 0\text{x}07 \\
0\text{x}03 & 0\text{x}01 & 0\text{x}05 & 0\text{x}04 & 0\text{x}08 & 0\text{x}06 & 0\text{x}07 & 0\text{x}0B \\
0\text{x}04 & 0\text{x}05 & 0\text{x}01 & 0\text{x}03 & 0\text{x}0B & 0\text{x}07 & 0\text{x}06 & 0\text{x}08 \\
0\text{x}05 & 0\text{x}04 & 0\text{x}03 & 0\text{x}01 & 0\text{x}07 & 0\text{x}0B & 0\text{x}08 & 0\text{x}06 \\
0\text{x}06 & 0\text{x}08 & 0\text{x}0B & 0\text{x}07 & 0\text{x}01 & 0\text{x}03 & 0\text{x}04 & 0\text{x}05 \\
0\text{x}08 & 0\text{x}06 & 0\text{x}07 & 0\text{x}0B & 0\text{x}03 & 0\text{x}01 & 0\text{x}05 & 0\text{x}04 \\
0\text{x}0B & 0\text{x}07 & 0\text{x}06 & 0\text{x}08 & 0\text{x}04 & 0\text{x}05 & 0\text{x}01 & 0\text{x}03 \\
0\text{x}07 & 0\text{x}0B & 0\text{x}08 & 0\text{x}06 & 0\text{x}05 & 0\text{x}04 & 0\text{x}03 & 0\text{x}01
\end{pmatrix} \tag{14}
$$

We give an experiment for 12-round *chosen-key* distinguisher of section 3.2 to find a pair that matches the 12-round differential pattern of Fig. 4a by using our Alg 1. We get a pair of plaintexts:

$$P_1 = (\text{1f 17 7f 72 7a f5 37 53, 5f f4 d9 23 59 e0 e6 75}),$$

$$P_2 = (\text{8a b5 11 89 23 29 49 9f, a1 9e 90 58 02 e8 fa 25}),$$

under $key = (\text{69 e4 4a 60 1e ea 50 20, 0a 3b 81 ae ad 3a 79 bc})$ (all the numbers are in hexadecimal). The corresponding differential of the 12-round reduced Camellia-MDS is listed in Tab. 2, which follows the differential pattern of Fig. 4a. We do not give a collision attack experiment because the complexity is infeasible under our computation resource.

Table 2: Differential of the Experiment Pair for 12-Round *Chosen-Key* Distinguisher

	Input Differences of Each Round	
1st Round	95 a2 6e fb 59 dc 7e cc	fe 6a 49 7b 5b 08 1c 50
2nd Round	32 00 00 00 00 00 00 00	95 a2 6e fb 59 dc 7e cc
3rd Round	00 00 00 00 00 00 00 00	32 00 00 00 00 00 00 00
4th Round	32 00 00 00 00 00 00 00	00 00 00 00 00 00 00 00
5th Round	02 06 08 0a 0c 10 16 0e	32 00 00 00 00 00 00 00
6th Round	a9 00 00 00 00 00 00 00	02 06 08 0a 0c 10 16 0e
7th Round	00 00 00 00 00 00 00 00	a9 00 00 00 00 00 00 00
8th Round	a9 00 00 00 00 00 00 00	00 00 00 00 00 00 00 00
9th Round	02 06 08 0a 0c 10 16 0e	a9 00 00 00 00 00 00 00
10th Round	51 00 00 00 00 00 00 00	02 06 08 0a 0c 10 16 0e
11th Round	00 00 00 00 00 00 00 00	51 00 00 00 00 00 00 00
12th Round	51 00 00 00 00 00 00 00	00 00 00 00 00 00 00 00
13th Round	a2 fb b2 10 eb 79 82 49	51 00 00 00 00 00 00 00

†: all the numbers are in hexadecimal.

6   Conclusion

In this paper, we give an answer to the open problem proposed by Sasaki in [Sas12] and show *chosen-key* scenario works well in the study of Feistel schemes. By using the *rebound-attack* technique and the available degrees of freedom in the key, we introduce 11-round collision attacks on two-block MMO/MP hash functions with Feistel-SP block ciphers. These improve previous best works by two rounds. Besides, 12-round *chosen-key* distinguishers are also presented.

Due to the development of industry, the lightweight cryptography applied to resource-restricted environment becomes more and more popular. If one needs both block cipher and hash function, then using a block cipher to construct a hash function can minimize the design and implementation cost. So the security analysis of these applications is immediately needed. This paper presents some results on the generic Feistel-SP block cipher used in hashing mode. However, it is far from enough. There are many works needed to be done, such as analysis on hash function with SPN block cipher, or many standardized primitives.

7   Acknowledgments

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