On the Security Margin of TinyJAMBU with Refined Differential and Linear Cryptanalysis

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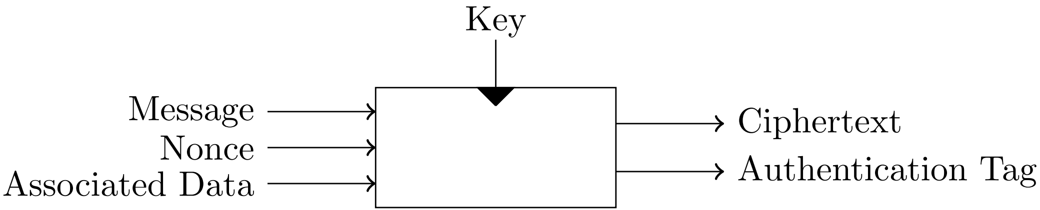
High-level Description - AEAD



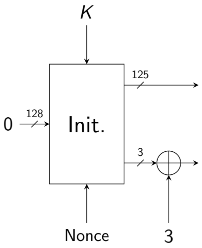
- Designed by Hongjun Wu and Tao Huang
- A small variant of JAMBU [WH15]
- A family of AEAD schemes
- Currently a Round-2 candidate in NIST LWC

Table: Security goals of TinyJAMBU with unique nonce

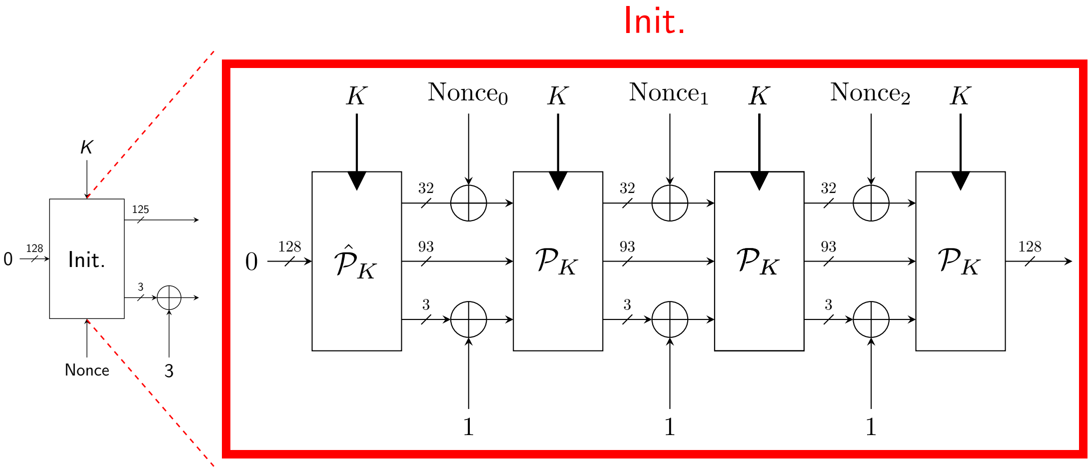
Version	Encryption	Authentication
TinyJAMBU-128	112-bit	64-bit
TinyJAMBU-192	168-bit	64-bit
TinyJAMBU-256	224-bit	64-bit

- WH15 - JAMBU Lightweight Authenticated Encryption Mode and AES-JAMBU. Submission to CAESAR, 2015

Step 1: Initialization

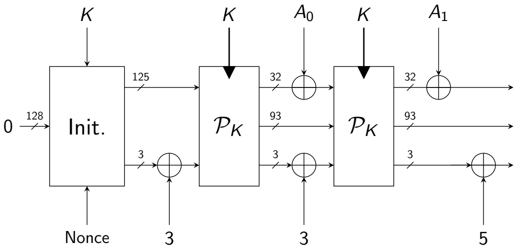


Inside Init. (Key Setup + Nonce Setup)

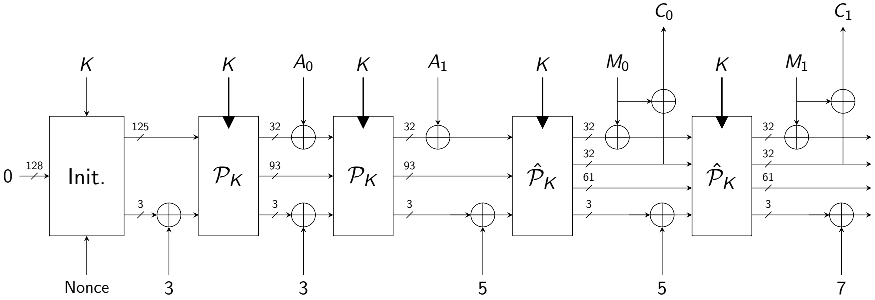


$\mathcal{P}_K, \hat{\mathcal{P}}_K$ → Keyed Permutations

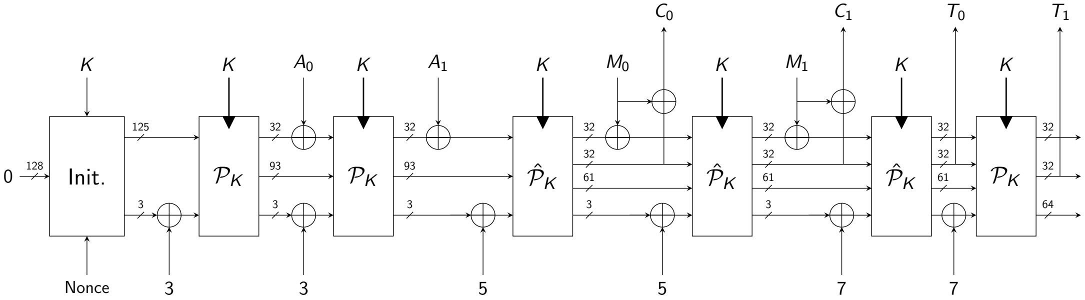
Step 2: Associated Data Processing



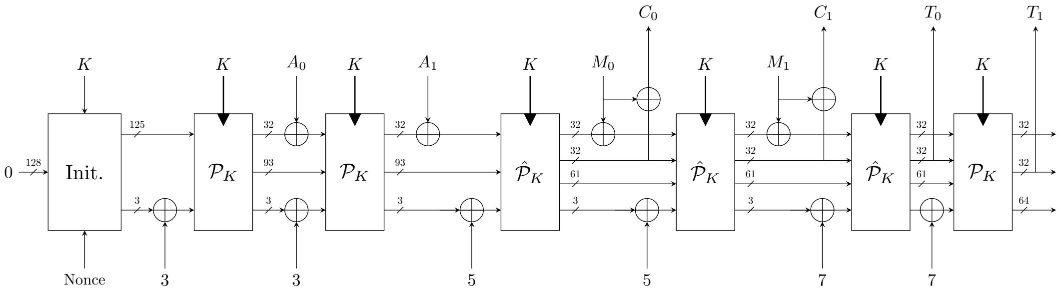
Step 3: Encryption



Step 4: Finalization



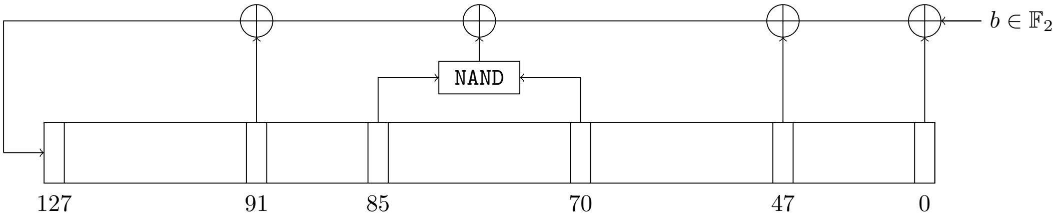
The Three Variants of TinyJAMBU



AEAD	Sizes in bits				# of rounds	
	State	Key	Nonce	Tag	$\mathcal{P}_K$	$\hat{\mathcal{P}}_K$
TinyJAMBU-128	128	128	96	64	384	1024
TinyJAMBU-192	128	192	96	64	384	1152
TinyJAMBU-256	128	256	96	64	384	1280

- Note: The number of rounds of $\hat{\mathcal{P}}_K$ is much **larger** than that of $\mathcal{P}_K$
- Used in Key Setup and Encryption

- NLFSR based keyed-permutation
- Computes only a single NAND gate as a non-linear component per round



Previous Cryptanalysis and Research Challenges

Strategy

Counts the number of **active AND** gates to find differential and linear trails with the minimum of such active gates by MILP

Why is this insufficient? → **Fast but inaccurate**

- Ignores the correlation between multiple AND gates which can impact probabilities of the differential or linear trails [KLT15, AEL+18]
- Designers have ignored effect of differentials which can amplify the probabilities of the trails [AK18]
- For linear cryptanalysis designer only analyzed internal permutation assuming access to all input bits

- KLT15 - Kölbl et al. Observations on the SIMON block cipher family. CRYPTO 2015
- AEL+18 - Ashur et al. Cryptanalysis of MORUS ASIACRYPT 2018
- AK18 - Ankele and Kölbl. Mind the Gap - A Closer Look at the Security of Block Ciphers against Differential Cryptanalysis. SAC 2018

A Note on Existing Literature on MILP Modeling

- Techniques exists to evaluate the exact probability by limiting the search space to only valid trails [SHW+15a, SHW+15b]

What is the issue? → **Accurate but too slow**

- Such models involve too many variables and constraints
- Cannot be solved in practical time
- Good for verifying the validity of a given trail
- Not so efficient to find optimal ones [SHW+15a]

- SHW+15a - Sun et al. Constructing mixed-integer programming models whose feasible region is exactly the set of all valid differential characteristics of SIMON. ePrint 2015
- SHW+15b - Sun et al. Extending the applicability of the mixed- integer programming technique in automatic differential cryptanalysis. ISC 2015

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Our Motivation: Strike a good balance of efficiency and accuracy while modeling

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Our Contributions

Identifying Issues With Simple MILP Model

What happens in the simple model?

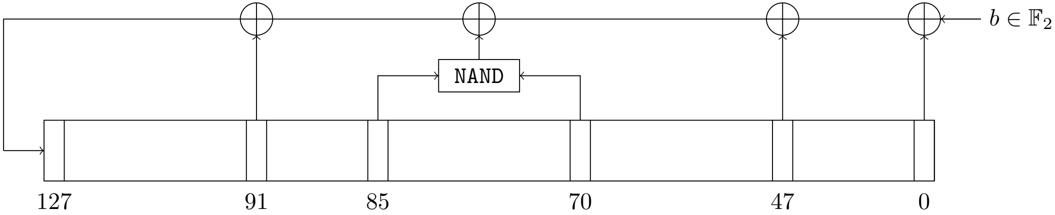
If there is a difference on at least one of the two input bits, the output of the AND gates has a difference with probability 2^{-1} or does not with probability 2^{-1}

- It considers independently every AND gate and
- Treats every AND gate in the same way

Table: Restrictions on the values of a and b in $a \cdot b = z$ when $\Delta z = 1$.

Δa	Δb	$\Delta z = 1$ iff
0	0	Never
0	1	$a = 1$
1	0	$b = 1$
1	1	$a = b$

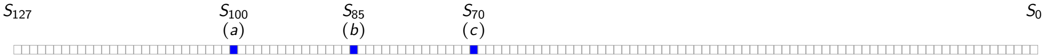
Simple model fails to capture these restrictions



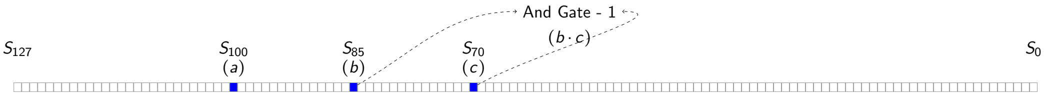
Main Observation

The same value, as it is shifted, will enter twice in two different AND gates.

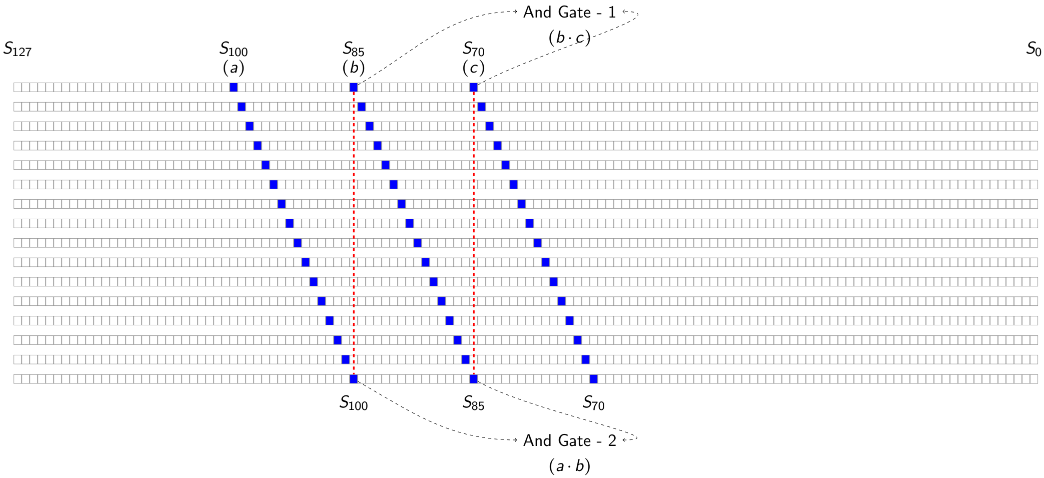
The Internal State $(S_{127}, \cdots S_0)$



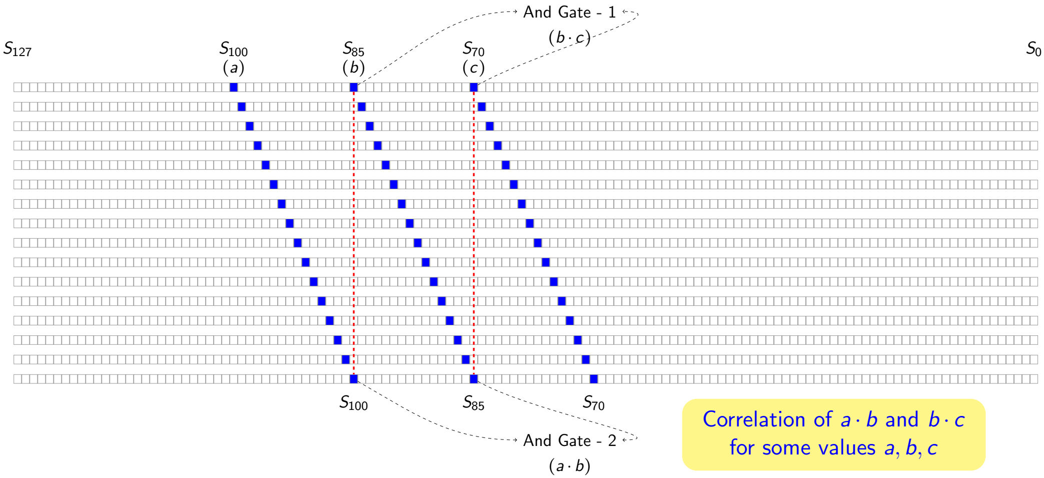
S_{85} Enters AND gate Twice (First: $b \cdot c$)



After 15 rounds (Second: $a \cdot b$)

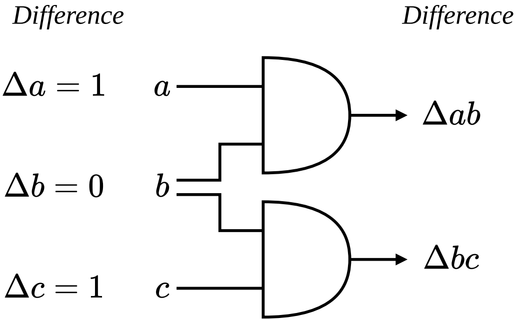


First Order Correlations

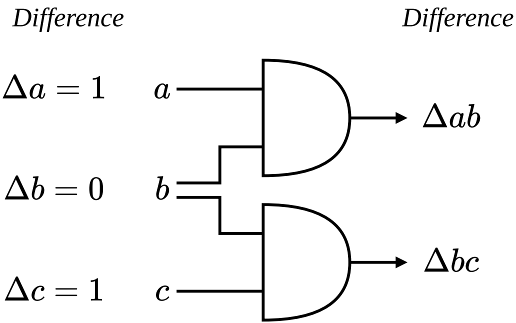


Correlation of $a \cdot b$ and $b \cdot c$ for some values a, b, c

Dependency of two AND gates



Dependency of two AND gates

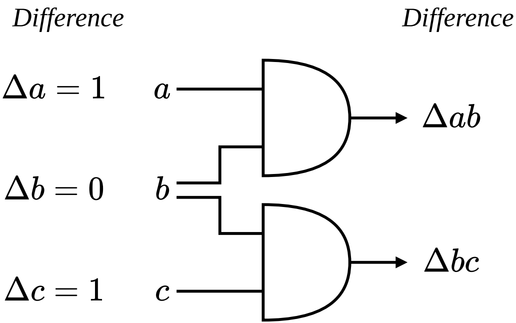


Case-1: $b = 0$

$\Delta ab = \Delta bc = 0$

Probability $= 2^{-1}$

Dependency of two AND gates



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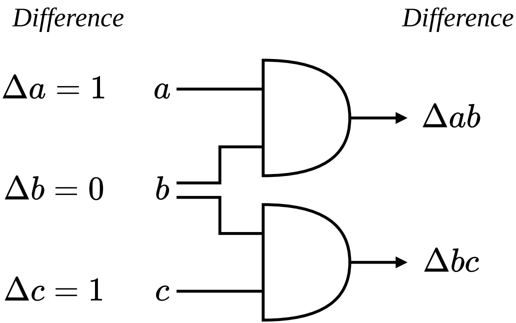


Case-1: $b = 0$

$\Delta ab = \Delta bc = 0$

Probability $= 2^{-1}$

Case-2: $b = 1$

$\Delta ab = \Delta bc = 1$

Probability $= 2^{-1}$

In this scenario — **Refined model**

- Forces that both differences jointly propagate, or not, and
- Only counts this as a **single** active gate.

MILP model variables:

- d_a modelizes Δa
- d_{ab} modelizes Δab
- γ_{abc} indicates if there's a correlation between the two AND gates ab and bc.

Finally

Subtract all values γ_{abc} in the objective function to only count this **once**, whereas the simple model would count two active gates.

- It adds **additional** constraints on top of the simple model
- All chained AND gates are recorded

Example Recorded Chains - $\{(d_{ab}, d_a, d_b), (d_{bc}, d_b, d_c), \ldots\}$

Then for all consecutive couples $\left((d_{ab}, d_a, d_b), (d_{bc}, d_b, d_c)\right)$ the following constraint is added:

$$\begin{aligned} \gamma_{abc} &= d_a \overline{d_b} d_c \\ d_{ab} - d_{bc} &\leq 1 - \gamma_{abc} \\ d_{bc} - d_{ab} &\leq 1 - \gamma_{abc} \end{aligned}$$

Differential Cryptanalysis

Trail Types in TinyJAMBU Submission Doc

- Designers searched for the differential trail that has the minimum number of active AND gates in the **simple** model

Type 1: Input differences only exist in the 32 MSBs. No constraint on the output.

Type 2: No constraint on the input. Output differences only exist in the 32 MSBs.

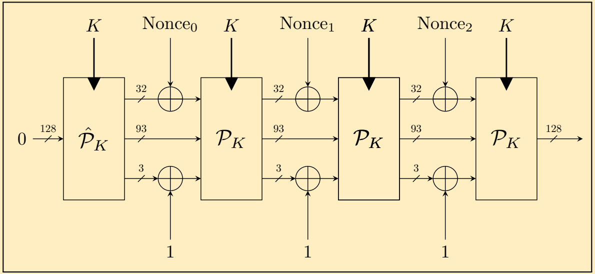
Type 3: Both of the input and output differences only exist in the 32 MSBs.

Type 4: No constraint.

Designers Claim — Proven **Wrong** in Refined Model

- Max. probability of the 384-round trail of Type 3 is 2^{-80}
- Max. probability of the 320-round characteristic of Type 4 is 2^{-13}

Forgery for TinyJAMBU Mode



- Attack the nonce setup or
- The associated data processing
- Recall $\mathcal{P}_K \rightarrow$ 384 Rounds
- Use Type 3 trails

Exploiting $(\Delta_i \| 0^{96}) \xrightarrow{\mathcal{P}_K} (\Delta_{i+1} \| 0^{96})$ with probability p

- Also makes the case for MAC reforgeability [BC09]
- **Unlike** designers we also look at cluster of multiple trails

- BC09 - Black and Cochran. MAC reforgeability. FSE 2009

Observations on Full 384 Rounds

- Found contradiction for simple model
- Refined model reports 88 active AND gates
- 14 couples are correlated
- Prob. $= 2^{-(88-14)} = 2^{-74}$

Input:	$\Delta S_{127..0}$	01004800	00000000	00000000	00000000
	$\Delta S_{255..128}$	81044c80	24080304	d9200000	22090000
	$\Delta S_{383..256}$	81004082	00010200	83000010	26090240
Output:	$\Delta S_{511..384}$	81004082	00000000	00000000	00000000

103 distinct differential trails

Overall Differential Prob. $= 2^{-70.68}$

Probability	2^{-74}	2^{-75}	2^{-76}	2^{-77}	2^{-78}	2^{-79}	2^{-80}
# Trails	1	5	9	14	20	24	30

Differential Cryptanalysis of 338 Rounds

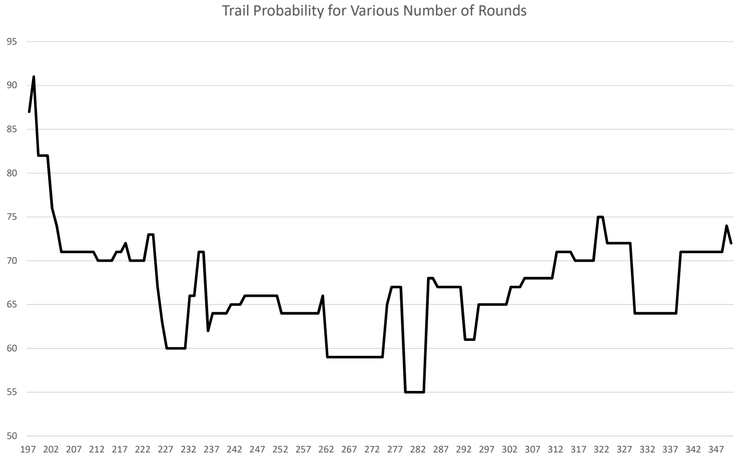
- Find largest number of rounds with security less than 64 bits
- Trail found with 76 active AND gates
- Correlation of two AND gates occurs 12 times
- Prob. $= 2^{-(76-12)} = 2^{-64}$

Input:	$\Delta S_{127..0}$	80104912	00000000	00000000	00000000
	$\Delta S_{255..128}$	00104c12	24800628	91000810	40092240
	$\Delta S_{383..256}$	00000000	00000200	81040000	04010200
Output:	$\Delta S_{465..338}$	00802041	00000000	00000000	00000000

24 distinct differential trails — **Overall Differential Prob.** $= 2^{-62.68}$

Probability	2^{-64}	2^{-66}	2^{-67}	2^{-68}	2^{-69}	2^{-70}	2^{-71}	2^{-72}
# Trails	1	2	4	4	4	5	4	4

Interesting Observation for Type 3



Vertical axis denotes the score. Horizontal axis denotes #rounds

Unrestricted Differentials

- No restriction on the input or output
- Type 4 as per `TinyJAMBU` submission document

Rounds	192	320	384
Designers (Simple)	4	13	-
Ours (Refined)	4	12	19

Type 4 Found with refined model **Prob.** $= 2^{-19}$

Input:	$\Delta S_{127..0}$	80000000	20010000	00000092	00000000
	$\Delta S_{255..128}$	00000000	20000000	00004000	00000004
	$\Delta S_{383..256}$	00000000	20000000	00000000	00000000
Output:	$\Delta S_{511..384}$	81020000	20001000	00004080	00000004

- Trails experimentally **verified**[1] with conforming pairs

[1] https://github.com/c-i-p-h-e-r/refinedTrailsTinyJambu

Attacks for the Underlying Permutation

Partly Restricted Differentials

- Type 1 (Input restricted)

Rounds	256	320	384	448	512
Designers (Simple)	22	33	45	55	68
Ours (Refined)	20	29	41	51	64?

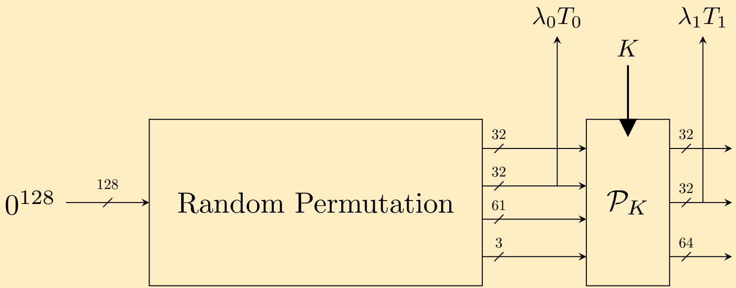
- Type 2 (Output restricted)

Rounds	384	512
Designers (Simple)	28	47
Ours (Refined)	28	47

- Note Type 1 Score is improved for all rounds
- Combining Type 1 and 2 for forgery (384 Rounds) as suggested in submission document
  - Designers → 2^{-73}
  - Ours → 2^{-69}

Linear Cryptanalysis

Linear trails of TinyJAMBUcarrying the correlation of the tag



- We can adapt the **same idea** of correlated AND gates to refine our model to look for better linear approximations

Refined Analysis for Partially Restricted Keyed Permutation

- The best linear trails were consistently having **no** correlated gates
- Score of the best linear trail with unrestricted input, restricted output:

Rounds	256	320	384	448	512
Designers	12	16	22	26	29
Ours (Refined)	10	15	22	27?	46?

Linear Bias of the Tag in the AEAD Setting

- Bias 2^{-41} optimal linear trail for 384 rounds found with the refined model
- Does not contradict the authors' claims

Input:	$mS_{127..0}$	00000000	41100081	00000000	00000000
	$mS_{255..128}$	00408000	41120491	02008024	08000088
	$mS_{383..256}$	30c80024	41804890	00449144	80000089
Output:	$mS_{511..384}$	00000000	00022890	00000000	00000000

Summary

- First 3rd-Part Cryptanalysis of `TinyJAMBU`
  - Reveals structural weakness of the mode ← Multi-block nonce/tag processing
- Refined model efficiently finds highly accurate differential and linear trails
- With the refined model, we found
  - A forgery attack with complexity $2^{62.68}$ on 338 rounds
  - A differential trail with probability $2^{-70.68}$ for the full 384 rounds
- Security margin of `TinyJAMBU` is smaller than originally expected
  - 12% with respect to the number of unattacked rounds
  - Less than 8 bits in the data complexity for the full rounds.
- Refined model for the linear cryptanalysis found the better bias for some number of rounds.
- One simple solution would be to increase the number of rounds of the small version, $\mathcal{P}_K$ from 384 to 512 rounds.
- Using the refined model may lead to a better choice of tap positions with respect to DC/LC

Thank You

Image Source: Google

Work **initiated** during group discussion sessions of ASK 2019, Japan

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The source code for finding conforming pairs and the MILP trails search can be found here `https://github.com/c-i-p-h-e-r/refinedTrailsTinyJambu`